Converges by AST

18 $\sum_{n=2}^{\infty} \left|(-1)^{n+1}\frac{1}{\ln n}\right| = \frac{-1}{\ln 2}+\frac{1}{\ln 3}-\frac{1}{\ln 4}+\ldots$

1. signs alternate ✓
2. $|a_{n+1}|<|a_n|$ ✓✓
3. $a_n \to 0$ ✓✓✓

$\sum_{n=2}^{\infty}\frac{1}{\ln n} > \sum \frac{1}{n^2}$

converges

$\sum_{n=2}^{\infty}\frac{1}{\ln n} > \sum \frac{1}{n}$ p series, $p=2$

diverges diverges

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21. $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\cancel{\ln n}}{2\cancel{\ln n}} = -\frac{1}{2}+\frac{1}{2}-\frac{1}{2}\ldots$

series diverges by n^{th} term test

19 $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{10^n}{n^{10}}$ diverges by n^{th} term test

$\lim_{n\to\infty}\frac{10^n}{n^{10}} = \lim_{n\to\infty}\frac{10^n \ln 10}{10\, n^9}$

$= \lim_{n\to\infty}\frac{10^n \ln 10 \cdot \ln 10}{10\cdot 9\, n^8}$

$= \lim_{n\to\infty}\frac{10^n(\ln 10)^{10}}{10!} = \infty$

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9.5b Alternating Series, Checking Endpoints

Alternating Series Test with remainder

$a_1 - a_2 + a_3 - a_4 \ldots a_n \,\}\ldots + \epsilon$

If ① signs alternate
② $|a_{n+1}|<|a_n|$
③ $a_n \to 0$

AST

then series converges

$\epsilon < |a_{n+1}|$ if $a_{n+1}>0$, underestimate
if $a_{n+1}<0$, overestimate

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Prove the alternating harmonic series is convergent but not absolutely convergent

Converges by AST

$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\ldots$

converges conditionally

1. signs alt. ✓
2. $|a_{n+1}|<|a_n|$ ✓✓
3. $a_n\to 0$ ✓✓✓

abs. conv

$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{n}\right| = 1+\frac{1}{2}+\frac{1}{3}\ldots$ harmonic diverges

p series $p=1$

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Conditional Convergence

$\sum a_n$ converges — probably use AST

$\sum |a_n|$ diverges — use some other test

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Find the interval of convergence for the following series. Be sure to check the endpoints.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n}$$

$x^{2(n+1)}$

1) ratio test (abs. conv.)

$$\lim_{n\to\infty} \left| \frac{x^{\cancel{2n+2}}}{2n+2} \cdot \frac{2n}{\cancel{x^{2n}}} \right|$$

$$\lim_{n\to\infty} \left| x^2 \cdot \frac{2n}{2n+2} \right| = \sqrt{x^2} < \sqrt{1}$$

$\frac{2n}{2n+2} \to 1$

$L < 1$

$|x| < 1$

$-1 \le x \le 1$

2) $x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \ldots$ converges by AST

3) $x = -1$ Same as 2)

$[-1, 1]$ ioc

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$$\sum_{n=0}^{\infty} \frac{(10x)^n}{n!}$$ find ioc

$$\lim_{n\to\infty} \left| \frac{(10x)^{\cancel{n+1}}}{(n+1)!} \cdot \frac{\cancel{n!}}{\cancel{(10x)^n}} \right| = 0 \quad \text{for all } x$$

$L = 0$

ioc = all x

$(-\infty, \infty)$

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$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2n}$$ find ioc

$$\lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{2n+2} \cdot \frac{2n}{(x-3)^n} \right| = \lim_{n\to\infty} \left| (x-3) \frac{2n}{2n+2} \right| = |x-3|$$

$L = |x-3| < 1$

$-1 < x-3 < 1$

$2 \le x < 4$

$x = 2$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ Converges by AST

$x = 4$ $\sum_{n=1}^{\infty} \frac{1}{2n}$ harmonic diverges p-series $p = 1$

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